CONAN: Contrastive Fusion Networks for Multi-view Clustering

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Abstract—With the development of big data, deep learning has made remarkable progress on multi-view clustering. Multi-view fusion is a crucial technique for the model obtaining a common representation. However, existing literature adopts shallow fusion strategies, such as weighted-sum fusion and concatenating fusion, which fail to capture complex information from multiple views. In this paper, we propose a novel fusion technique, entitled contrastive fusion, which can extract consistent representations from multiple views and maintain the characteristic of viewspecific representations. Specifically, we study multi-view alignment from an information bottleneck perspective and introduce an intermediate variable to align each view-specific representation. Furthermore, we leverage a single-view clustering method as a predictive task to ensure the contrastive fusion is working. We integrate all components into an unified framework called CONtrAstive fusion Network (CONAN). Experiment results on five multi-view datasets demonstrate that CONAN outperforms state-of-the-art methods. Our source code will be available soon at https://github.com/guanzhou-ke/conan.

Index Terms—Clustering, multi-view fusion, contrastive learn-

I. Introduction

In real-world scenarios, data are frequently represented as multiple views or modalities. For example, various descriptors may characterize an image, such as SIFT [1] or histograms of oriented gradients (HoG) [2]. Multi-view learning is a promising way to obtain good representations from multiples views. Due to a large amount of unlabeled data in the real world, unsupervised multi-view learning, e.g., multi-view clustering, has attracted the attention of the machine learning community.

The significant difference between multi-view and singleview clustering is that the former needs to extract the valuable representation shared from multiple views. It means that fusion is a crucial technique for multi-view clustering. Previous works focus on some simple fusion strategies [3], such as weighted-sum fusion [4], [5] and concatenating fusion [6]–[8]. These methods are difficult to capture high-level semantics from multiple views because they ignore the relationship of common representations and view-specific representations during the fusion process. Another fusion technique, neural network fusion, focuses on capturing the latent structure shared from multiple views [9], [10]. It can capture a more complex structure than the former but could harm the view-specific representations during the fusion process, called over-fusion. To against over-fusion, they employ Generative Adversarial Nets (GAN) to the constraint condition [11] to maintain view-specific representations [4], [6], [12]. However, the trick brings an underlying risk: inconsistent optimization objective. We observe that clustering focuses on extracting class-level information, but GAN aims to reconstruct original space from pixel-level information. It leads to the network being forced to learn some useless information for predicting labels during the optimization process.

In this paper, we identify the major challenge in multiview clustering fusion: can we extract harmonious common representations from each view under maintaining the viewspecific representations? The simple idea is that we could introduce a new representations space to align each viewspecific representation. For maintaining the independent representation space of each view, we block the interaction of view-specific representations. We study this idea from an information bottleneck perspective [13]. Inspired by previous works [14], [15], we can introduce an additional intermediate variable (representation space) to align each view-specific representation. We found that the contrastive learning method can be a suitable alignment method. Hence, we leverage the contrastive learning method [16] to improve multi-view fusion called contrastive fusion, discussed in section III.

However, we found that the contrastive fusion component cannot work well because it lacks the objective of extracting task-relevant information. In other words, it cannot identify

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what a good direction for alignment view-specific representations is. According to the suggestion of [15], we observe that a suitable predictive task, e.g., the single-view clustering method, can help the contrastive fusion component to extract harmonious common representations, discussed in section III-C. To this end, we integrate these components into a unified framework, namely CONtrAstive fusion Network (CONAN). The main contributions of this paper are summarized as follows:

- We propose a novel fusion technique, which extracts common representation from view-specific representations and preserves the independence of each view space.
- 2) We integrate all components into an unified framework, as shown in Fig.1. The framework we proposed is more light-weighted than previous works [4], [6].

II. RELATED WORK

In this section, we give a brief summary of the existing work on contrastive learning and multi-view clustering.

A. Contrastive Learning

Contrastive learning [16] is one of the unsupervised methods that have developed rapidly in recent years, especially in visual representations [17]–[19]. Contrastive learning aims to group the positive sample pairs and repulse the negative sample pairs, where the positive pair denotes the different augmentation views of an image. The negative pair is the augmentation view of other images. There is some literature to study the relation of between contrastive learning and multi-view learning from information bottleneck theory [14], [15], [20]. In [15], the authors proposed contrastive multi-view coding (CMC), which obtains robust representations by the multiple views augmentation of an image (more than two views). They found that the robustness of representations can be improved by increasing the number of views. On the other hand, the literature [14], [20] found that it can help extract consistent information by applying contrastive methods on multiple views. Based on the previous works, we introduce an intermediate variable and apply a contrastive method between this variable and each view-specific representations, called contrastive fusion. Intuitionally, the view-specific representations get aligned in the new representation space. The advantage of this is that it does not damage the view-specific representations and can achieve multi-view fusion.

B. Multi-view Clustering

The multi-view clustering taxonomy involves two categories: traditional methods and deep learning methods. Traditional methods rely on statistical theory, e.g., multi-kernel learning methods [21], [22], where the characteristics of different views can be non-linearly mapped into a common representation. Nevertheless, these methods require selecting the appropriate kernel for each view carefully. Subspacebased methods [23]–[25] project the multi-view data into a low-dimensional shared subspace to obtain the standard representation. There are also methods exploiting non-negative

matrix factorization [26] and graph-based models [27]. These statistical models have a common drawback, as they cannot extract complex structures within the data. Therefore, there have been many deep methods combined with traditional methods, such as [9], [28], [29]. These extract higher semantic features through neural networks and then apply on these features statistical models to extract a common representation.

In recent years, the complete deep learning methods have been proposed, such as [4], [6]–[8], [12]. These methods have a more vital ability to express a data structure, obtaining higher performance on many benchmark datasets. The literature [4], [6], [12] adopts GAN as multi-view fusion constraint to prevent over-fusion. It may be beneficial in generative tasks, but degenerate solutions will appear in multi-view clustering tasks. Using GAN constraints seems to learn discriminative features, but it still requires the network to learn pixel-level features. This phenomenon we call inconsistent optimization objective, we mentioned before. Compared with previous methods, our proposed contrastive fusion does not require the network to learn pixel-level features, and it belongs to the same kind as clustering, i.e., discriminative model.

III. METHOD

Our goal is to group a set of n data points consisting of V views $\mathcal{D} = \{\mathbf{X}^1, \mathbf{X}^2, ..., \mathbf{X}^V\}$ into c clusters, where $\mathbf{X}^v \in \mathbb{R}^{n \times d_v}$ denotes the samples of the dimension d_v from the v-th views.

A. Networks Architecture

The proposed approach, CONAN, consist of V view-specific encoder networks $e_v(\cdot)$, a fusion network $f(\cdot)$, a small neural network projection head $p(\cdot)$, and a clustering head $g(\cdot)$, as illustrated in Fig.1.

- View-specific encoder networks $e_v(\cdot)$ that extracts view-specific representations(vectors) h from each view data point, i.e., $\mathbf{h^v} = e_v(\mathbf{X_v})$. CONAN allows various forms of the network architecture as encoder, e.g., Fully-Connected Networks (FCNs) or Convolution Neural Networks (CNNs), according to the data type. In practice, we adopt CNNs to process image data and FCNs for vector-liked data.
- A fusion network f(·): it fusions concatenating vectors \$\bar{\mathbf{h}} = cat(\mathbf{h}^1, ..., \mathbf{h}^v)\$ to obtain the common representation \$\mathbf{z} = f(\bar{\mathbf{h}})\$. We adopt two fully-connected layers with ReLU as fusion network. The major advantage of our fusion network is that can fit better any complex function [30] than the weighted-sum method.
- A projection head p(·): it maps representations to space where contrastive loss is applied. We adopt the same setting in previous work [17] to build our projection head. We discuss its details in the next section.
- A clustering head $g(\cdot)$: it assigns each data point to a compact space according to z, in other words, we can obtain cluster labels from it, i.e., Y = g(z). This component could adopt any single-view clustering methods,

such as Deep Divergence Clustering (DDC) [31] or Deep Embedding Clustering (DEC) [32].

In summary, our proposed framework, CONAN, is more light and flexible than the previous AE-Based [9], [29], [33], or GAN-Based [4], [6], [12] framework. Furthermore, CONAN adapts any online clustering methods thanks to the common representation **z** we obtained by contrastive fusion network.

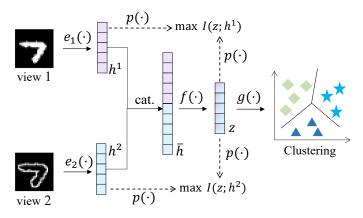


Fig. 1. Illustration of CONAN. It consist of V view-specific encoder networks $e_v(\cdot)$ to obtain view-specific representations $\mathbf{h^v}$, a fusion network $f(\cdot)$ to obtain the common representation \mathbf{z} via fusion the concatenating vectors $\overline{\mathbf{h}}$, a small neural network projection head $p(\cdot)$ to map representations to space where contrastive loss is applied, and a clustering head $g(\cdot)$ to obtain soft assignment $Y = g(\mathbf{z})$. $I(\cdot; \cdot)$ represent mutual information.

B. Contrastive Fusion

Our goal is to extract the task-relevant information (consistency) shared between view-specific representations *h* and discard the task-irrelevant information. For convenience, we discuss two views situation, but it can be easily extended to multiple view cases. According to the conclusion of previous works [14], [15], [20], we can formula our goal from the information bottleneck perspective as the following definition:

$$I(\mathbf{X}^1; \mathbf{h}^2) = I(\mathbf{X}^2; \mathbf{h}^2) - I(\mathbf{X}^2; \mathbf{h}^2 | \mathbf{X}^1)$$
(1)

$$I(\mathbf{X}^2; \mathbf{h}^1) = \underbrace{I(\mathbf{X}^1; \mathbf{h}^1)}_{\text{task-relevant}} - \underbrace{I(\mathbf{X}^1; \mathbf{h}^1 | \mathbf{X}^2)}_{\text{task-irrelevant}}$$
(2)

It is benefit for extracting the task-relevant information when both maximizes (1) and (2), but we cannot maximize them directly in unsupervised setting. According to the conclusion of the literature [14], however, we can minimize the second term in (1) and (2) to maximize the lower bound on the mutual information $I(\mathbf{X}^1; \mathbf{h}^2)$ and $I(\mathbf{X}^2; \mathbf{h}^1)$. We discuss our strategy to minimize the conditional mutual information in next section.

According to above theory, we introduce an intermediate variable called \mathbf{z} which can both maximize $I(\mathbf{z}; \mathbf{h}^1)$ and $I(\mathbf{z}; \mathbf{h}^2)$. Hence, we can rewrite the original definition as following:

$$I(\mathbf{X}^1; \mathbf{z}) = I(\mathbf{X}^2; \mathbf{z}) - I(\mathbf{X}^2; \mathbf{z} | \mathbf{X}^1)$$
(3)

where \mathbf{z} is the common representation what we need. We call the process of maximization $I(\mathbf{z}; \mathbf{h}^1)$ and $I(\mathbf{z}; \mathbf{h}^2)$ as contrastive fusion, and then we focus on how to measure \mathbf{z} and each \mathbf{h} , i.e., $\max I(\mathbf{z}; \mathbf{h}^v)$. Inspired by contrastive learning methods [17], [18], we introduce cosine similarity to compute the score between \mathbf{z} and \mathbf{h}^v , i.e., $sim(\mathbf{z}, \mathbf{h}^v) = \mathbf{z}^T \mathbf{h}^v / \|\mathbf{z}\| \|\mathbf{h}^v\|$. Then, we extend the loss function in [17] to multiple views setting:

$$\mathcal{L}_{contrast}^{simclr} = -\sum_{v=1}^{V} log \frac{\exp(sim(p(z_i), p(h_j^v))/\tau)}{\sum_{k=1}^{2N} \mathbf{1}_{[k \neq i]} \exp(sim(p(z_i), p(h_k^v))/\tau)}$$
(4)

where $p(\cdot)$ is a projection head, (i,j) is a pair positive examples, $\mathbf{1}_{[k \neq i]} \in \{0,1\}$ denotes an indicator function evaluating to 1 if $k \neq i$, and τ represents a temperature parameter which we adopt optimal setting as $\tau = 0.1$ suggested by [17] in our experiments. According to the conclusion of [15], we can maximize $I(\mathbf{z}; \mathbf{h}^v)$ by minimization the contrastive loss.

On the other hand, there is another version contrastive objective which does not require negative samples, called SimSiam [18], as following:

$$\mathcal{L}_{contrast}^{simsiam} = \sum_{v=1}^{V} -\frac{1}{2} (sim(p(\mathbf{z}), q(\mathbf{h}^{v}) + sim(p(\mathbf{h}^{v}), q(\mathbf{z})))$$
(5)

where $q(\cdot)$ is a prediction head that consists of two fully-connected layers, note that the output vectors of $q(\cdot)$ can be treated as constant, which means that the output vectors of $q(\cdot)$ cannot receive any gradient during back-propagation. It seems that is the key trick of [18] to achieve state-of-the-art performance, called stop gradient. We study the impact of fusion via using two different objectives in section IV. According to the suggestion of ablation study in section IV, we adopt the default contrastive objective as (4) in CONAN.

C. Loss Function

In this section, we discuss our solution to minimize $I(\mathbf{X}^2; \mathbf{z}|\mathbf{X}^1)$ in (3). In an unsupervised setting, a common solution is to reconstruct the original data space what autoencoder does. The reconstruction try to minimize $I(\mathbf{X}; \mathbf{z}|\bar{\mathbf{X}})$, where $\bar{\mathbf{X}}$ is required to approximate the original data space \mathbf{X} . From a high level, the reconstruction or other downstream tasks, e.g., clustering or classification, can be roughly seem as a predictive learning task [15], [20]. According to these assumptions [15], [20], we can formula a general definition to instead $I(\mathbf{X}^2; \mathbf{z}|\mathbf{X}^1)$, as following:

$$I(\mathbf{X}^2; \mathbf{z}|T) = I(\mathbf{X}^2; \mathbf{z}|\mathbf{X}^1) + \epsilon_{info}$$
 (6)

where T denotes an alternative predictive task, and ϵ_{info} is the gap between T and the original predictive task. However, the choice of a suitable predictive task is still an open question in unsupervised learning. We empirically chose the online clustering task as the alternative predictive task to maximize (3). There are two common clustering tasks we used in

this work, called DDC [31] and DEC [32]. The DDC is a divergence-based clustering method that uses Cauchy-Schwarz divergence (CS-divergence) [34] to enforce cluster separability and compactness. Another one, DEC, is a KL divergence-based method that use the Student's t-distribution as a kernel to measure the similarity between hidden representation z_i and centroid μ_j . Next, we describe the loss function of DDC and DEC, respectively.

The loss function of DDC consists of three terms as shown in the following:

$$\mathcal{L}_{ddc} = \sum_{i=1}^{k-1} \sum_{j>i}^{k} \frac{\alpha_{i}^{T} \mathbf{K} \alpha_{j}}{\sqrt{\alpha_{i}^{T} \mathbf{K} \alpha_{i} \alpha_{j}^{T} \mathbf{K} \alpha_{j}}} + triu(AA^{T}) + \sum_{i=1}^{k-1} \sum_{j>i}^{k} \frac{m_{i}^{T} \mathbf{K} m_{j}}{\sqrt{m_{i}^{T} \mathbf{K} m_{i} m_{j}^{T} \mathbf{K} m_{j}}}$$
(7)

where A denotes the cluster assignment matrix, α_i is the i-th column of matrix A; \mathbf{K} denotes kernel similarity matrix computed by $\mathbf{k}_{ij} = \exp(-\|z_i - z_j\|^2/(2\sigma)^2)$, and σ is Gaussian kernel bandwidth, default as 0.15; $triu(AA^T)$ denotes the strictly upper triangular elements of AA^T ; $m_i j = \exp(\|a_i - e_j\|^2)$ where e_j is corner j of the standard simplex in \mathbb{R}^k .

The loss function of DEC as shown in the following:

$$\mathcal{L}_{dec} = \sum_{i=1}^{N} \sum_{j=1}^{k} p_{ij} log \frac{p_{ij}}{q_{ij}}$$
 (8)

where q_{ij} denotes the probability of assignment data point i to cluster j, and it compute by:

$$q_{ij} = \frac{(1 + \|z_i - \mu_j\|^2/\beta)^{-\frac{\beta+1}{2}}}{\sum_{j'}^{K} (1 + \|z_i - \mu_j'\|^2/\beta)^{-\frac{\beta+1}{2}}}$$

and

$$p_{ij} = \frac{q_{ij}^2 / \sum_i q_{ij}}{\sum_{j'} q_{ij'}^2 / \sum_i q_{ij'}}$$

According to the suggestion of DEC, we pass the whole dataset through the initialized view-specific encoders to get common representation \mathbf{z} and then perform standard k-means algorithm in \mathbf{z} to obtain k initial centroids $\{\mu_j\}_{j=1}^k$.

Finally, we integrate all components into an unified framework, namely CONAN. As default, we adopt DDC as the predictive task and (4) as the contrastive measurement during fusion view-specific representations, as shown in the following:

$$\mathcal{L}_{conan} = \mathcal{L}_{ddc} + \lambda \mathcal{L}_{contrast}^{simclr}$$
 (9)

where λ is a trade-off parameter to limit the contrastive loss term too large. As shown in section IV, we found that $\lambda=0.01$ is the optimal value for all experiments. Concretely, Algorithm 1 summarizes the proposed method.

Algorithm 1: CONAN pseudocode, PyTorch-style

```
e_v: v-th view-specific encoder.
 f: fusion module.
# q: clustering module.
 p: projection mlp.
 11: clustering loss.
# 12: contrastive loss.
for x1,...,xv in dataloader:
   # Step 1: obtain view-specific
   representations
   h1, ..., hv = e1(x1), ..., ev(xv)
   hs = cat(h1, ..., hv) # n-by-(v*d)
   # Step 2: fusion view-specific
   representations
   z = f(hs)
   # Step 3: soft assign z to k cluster
   logits = q(z)
   # Step 4: compute the joint loss.
   L = 11(z, logits) + \lambda \star C(z, hs)
    Step 5: back-propagate loss and
    update networks' parameters by Adam
   L.backward()
   update(e, f, g, p)
def C(z, hs) # compute contrastive loss
   hs = split(hs) # to list.
   zp = p(z) # projection.
   hps = [p(h) for h in hs]
    compute zq = q(z) and hqs = [q(h)]
   # for h in hs] if used simsiam loss.
   subloss = 0.
   # compute zp and each hp loss .
   for hp in hps:
      subloss += 12(zp, hp)
   return subloss
```

IV. EXPERIMENTS

A. Dataset

We evaluate CONAN and other competitors using five well-known multi-view datasets containing raw image and vector data, and report the dataset description in Table I. There are:

- Edge-MNIST (E-MNIST) [35], which is a large-scale and widely-used benchmark dataset consisting of 70,000 handwritten digit images with 28 × 28 pixels. The views contain the original digits and the edge-detected version, respectively. In unsupervised setting, we only need the training set to evaluate models.
- Edge-FMNIST (E-FMNIST) [36], which is a more complex dataset than standard MNIST consisting of 28 × 28 grayscale images of clothing items. We synthesize the second view by running the same edge detector used to create Edge-MNIST.
- COIL-20 [37], and COIL-100 [38], which depicts from different angles containing grayscale images of 20 items and RGB images of 100 items, respectively. We create a three-view dataset by randomly grouping the images for an item into groups of three.

 PASCAL VOC 2007 (VOC) [39]. We use the multi-view version provided by [40], which consists of GIST features and word frequency counts from manually tagged natural images.

TABLE I DATASET DESCRIPTION

Dataset	#samples	#view	#class	#shape*
Edge-MNIST	60,000	2	10	$1 \times 28 \times 28$
Edge-FMNIST	60,000	2	10	$1 \times 28 \times 28$
COIL-20	480	3	20	$1 \times 128 \times 128$
COIL-100	2,400	3	100	$3 \times 128 \times 128$
VOC	5649	2	20	512, 399

^{*}Single shape denotes that the input dimensionality is same for all views.

B. Baseline Models

We evaluate our framework to an extensive set of baseline models, which represent state-of-the-art for multi-view clustering: 1) Deep Canonical Correlation Analysis (DCCA) [28]; 2) Deep Canonically Correlated Auto-encoders (DCCAE) [29]; 3) Deep Multi-modal Subspace Clustering (DMSC) [9]; 4) Deep Adversarial Multi-view Clustering (DAMC) [6]; 5) End-to-end Adversarial attention network for Multi-modal Clustering (EAMC) [4]; 6) Autoencoder in autoencoder networks (Ae²-Nets) [33]. Furthermore, in order to demonstrate CO-NAN can improve the clustering performance compared with the single-view setting. We use the standard spectral clustering algorithm [41] to conduct on each view and the concatenated view, and then we report the best result in spectral clustering (SC-best).

C. Implementation Details

We implement CONAN and the competitor non-linear methods on the PyTorch [42] platform, running on CentOS Linux 7 utilizing two NVIDIA Quadro P5000 Graphics Processing Units (GPUs) with 16 GB memory size. We train our model for 100 epochs, using the Adam optimization technique [43] with default parameters, and using Cosine Annealing learning rate scheduler [44] with an initial learning rate of 0.001. We leverage the grid search to find an optimal trade-off parameter λ range from 10^{-3} to 1. We observe that sets too small value to λ would curb the fusion process, while too large is harmful to the predictive task. Hence, we set λ to 0.01 in all experiments. For Edge-MNIST and Edge-FMNIST, we adopt CNN, proposed in [9] as the view-specific encoder. We set the batch size of the training dataset to 128 and the dimensionality of hidden representation, i.e., h^v and z, to 288. For COIL-20 and COIL-100, we adopt AlexNet [45] as the view-specific encoder. We set the batch size to 24 and the hidden representation to 1024. Note that the suggestion of input shape in [45] is $3 \times 144 \times 144$, but we observe that it also work well on the COIL-(20/100) dataset. For VOC, we

TABLE II
CLUSTERING RESULTS ON FIVE BENCHMARK DATASETS.

Dataset	E-M	NIST	E-FM	NIST	COI	L-20	COIL	L-100	V	OC
Metrics	ACC	NMI	ACC	NMI	ACC	NMI	ACC	NMI	ACC	NMI
SC-Best $^{\alpha}$	-	-	-	-	-	-	-	-	.402	.411
$DCCA^{\beta}$.476	.443	.426	.419	-	-	-	-	.397	.425
DCCAE^{β}	.508	.475	.458	.436	-	-	-	-	.416	.443
DMSC	.653	.614	.524	.518	.651	.720	.483	.697	.541	.538
DAMC	.646	.594	.495	.487	.672	.729	.526	.701	.560	.552
Ae ² -Nets	.537	.463	.410	.392	.717	.812	.541	.753	.571	.571
EAMC	.668	.628	.541	.622	.690	.753	.518	.680	.607	.615
CONAN	.905	.861	.592	.552	.725	.803	.556	.774	.621	.621

 $^{^{\}alpha}$ We cannot apply spectral clustering on the dataset except VOC because the large-scale or high-dimensionality dataset occupies many computation resources. $^{\beta}$ DCCA and DCCAE can only apply on two views dataset.

adopt three fully-connected layers with ReLU as the view-specific encoder, i.e., x-512-512-256, where x denotes the input dimensionality. The batch size is set to 100. Due to the instability of unsupervised, we train CONAN 20 times and report the results from the run resulting in the lowest value of (9).

D. Evaluation Metrics

The clustering performance is measured using two standard evaluation matrices, i.e., clustering Accuracy (ACC) and Normalized Mutual Information (NMI), where a higher value indicates better performance. For further details on these evaluation metrics, the reader is referred to [46]. It should be noted that the validation process of the clustering methods involves only the cases where ground truth labels are available.

E. Compared with Baseline Models

We evaluate our model and baseline models on five well-known datasets and report the ACC and NMI results in Table II. The results illustrate that our model can have a significant improvement compared to baseline models. It is worth noting that the proposed method manages a performance improvement of 23.7%(0.905-0.668) and 23.2%(0.861-0.628) against the second-best method in terms of ACC and NMI metrics, respectively. We believe that the contrastive fusion we proposed outperforms the shallow fusion methods, such as concatenating (DAMC) and weight-sum (EAMC). On the other hand, our method is a friendly-resource algorithm compared with the self-expression layer in DMSC.

F. The Impact of Different Contrastive Loss

Contrastive measurement is a crucial component of our method. According to the conclusion of the state-of-the-art for contrastive learning, [18], there are two different type measurements, i.e., negative samples needed ($\mathcal{L}_{contrast}^{simclr}$) and positive samples only ($\mathcal{L}_{contrast}^{simsiam}$), that may lead to different

TABLE III
THE IMPACT OF DIFFERENT CONTRASTIVE LOSS ON E-FMNIST AND COIL-20/100 dataset.

	$\mathcal{L}_{contrast}^{simclr}$	$\mathcal{L}_{contrast}^{simsiam}$	ACC	NMI
IST	-	_	0.508	0.492
Ę	✓	_	0.592	0.552
E-FMNIST	-	\checkmark	0.603	0.551
-20	_	_	0.539	0.651
COIL-20	\checkmark	_	0.725	0.803
$\ddot{\circ}$	-	\checkmark	0.629	0.697
COIL-100	_	_	0.389	0.679
Ę	\checkmark	_	0.556	0.774
9	-	\checkmark	0.407	0.698

performance during the fusion process. Hence, we study the representative method of two kind measurements on E-FMNIST and COIL-20/100 dataset, as shown in Table III. The results illustrate that contrastive fusion can significantly improve the performance of clustering. Furthermore, we can see that there are no overwhelming results. Using $\mathcal{L}_{contrast}^{simclr}$ or $\mathcal{L}_{contrast}^{simsiam}$ in (9) achieve almost the same performance on the E-FMNIST dataset, but the former is almost entirely beyond the latter on the COIL-20/100 dataset.

Our conclusion is that because contrastive learning relies on the quality (data augmentation) and quantity of the dataset [47], both can achieve almost the same performance on the E-FMNIST dataset with sufficient quantity and quality. Conversely, if the data quality and quantity are insufficient, the performance depends on establishing negative samples, which is particularly obvious on COIL-100.

G. The Impact of Different Predictive Tasks

TABLE IV
THE IMPACT OF DIFFERENT PREDICTIVE TASKS ON E-FMNIST.

model	\mathcal{L}_{ddc}	\mathcal{L}_{dec}	ACC	NMI
Random	_	-	0.105	0.001
Ŋ	-	-	0.117	0.002
ONAN	\checkmark	_	0.592	0.552
ŏ	-	\checkmark	0.494	0.486

We study the different predictive tasks on E-FMNIST in order to understand the importance of a good predictive task to our model, the results as reported in Table IV. We found that it would be close to a random model if we removed the clustering head. It is in line with our conclusion in section III-C. In addition, there is a significant gap between \mathcal{L}_{ddc} and \mathcal{L}_{dec} . The results illustrate that a suitable predictive task is crucial to minimize (6). Nevertheless, the choice of predictive task is still an open question.

H. Visualization of The Contrastive Fusion

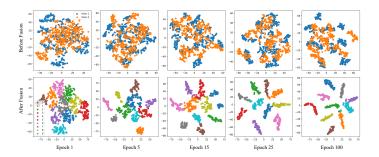


Fig. 2. Visualization of representations on the E-MNIST dataset before (first row) and after fusion (second row) using T-SNE at epoch 1st, 5th, 15th, 25th, and 100th.

We demonstrate the change of representations before and after contrastive fusion on the E-MNIST dataset, as shown in Fig.2. For convenience, we randomly choose 1K samples projected to 2-D using T-SNE [48]. We can see that the view-specific representations \mathbf{h}^1 and \mathbf{h}^2 always keep their unique characteristic. On the other hand, the common representation \mathbf{z} becomes more separate and compact with increasing the training epoch. The results illustrate that contrastive fusion can improve the clustering performance and does not harm the view-specific representation.

V. CONCLUSION

In this work, we proposed a contrastive fusion strategy that aligns view-specific representation into a new representation space (common representations). We apply the existing single-view online clustering method to help the contrastive fusion network extracting valuable information from multiple views. Experimental results on five real-world datasets demonstrate the contrastive fusion improves the quality of unsupervised representations.

ACKNOWLEDGMENT

This work was supported by Natural Science Foundation of Guangdong Province, China (No. 2020A1515011468; 2016A0303010003); Guangdong University Scientific Research Project, China (No. 2019KTSCX189; 2018KTSCX235; 2018WZDXM014); Joint Research and Development Fund of Wuyi University and Hong Kong and Macau (2019WGALH21).

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